The POLYHEDRON of MELENCOLIA § I
“polyhedron of Dürer”

INSTRUCTIONS for 3D
Installation “MELENCOLIA de”

by Yvo Jacquier
The polyhedron of Albrecht Dürer has been a long time a great enigma. Mathematicians have recently solved its proportions (especially the circum and the inscribed spheres), thanks to the discovery of sketches from the hand of the artist.

Two important points emerge:
- The form is organized by the Pentagram, that explains the presence of φ in the results.
- This solid is the missing element to the set of Plato to achieve the model of Kepler. It is the representation of the planet Earth. Not as an element among Water, Sky and Fire, but in the chain of Mercury and Saturn.
THE FACES

The six main faces are identical, and they take five points of a double pentagram. The solid is closed on the top and the bottom by two equilateral triangles (they are inverted to draw an hexagram), which are the mark of this truncated rhombohedron.

The precise measures and properties are further.
THE SYMBOLICAL MEANING

The original form is a cube, marked by the integer 4, which is inclined by the magic of φ to get a rhombohedron. This solid is crossing to sets of three faces, number that enhance the triangles. So the solid is also referring to 3. In this demonstration, the man born symbolically on Earth (4) and goes to heaven (3).

The account of the apexes gives the multiplication of 3 by 4, id est 12. If we consider the pentagrams, the hexagrams (with the two triangles), the eight faces, the eighteen edges, a very large part of the symbolic vocabulary is shown.

The Earth as a planet can not be resumed to one of the classical element (Earth, Water, Air, and Fire). All of them are present. So the solid which represents the Earth has to assume all through numbers.

A last, a very symbolical fact is signing this purpose : the two spheres (in and circum ones) are showing the angle of the Earth on the ecliptic, which is called obliquity. The inscribed sphere touches the wide faces at the middle of their structural circle (the middle of their two pentagrams, that we saw at the begining). This points are called Ω.

The line joining the top of the great sphere and Ω is very close to 23,5° and to the obliquity. The precision is 4,5% of degree in the actual state of the Earth's position.

In his writings, Dürer uses about the man the image of the block of carved stone, in the goal to affirm the role, even necessity, of education. The Polyhedron represents an inhabited Earth, by evolved men, able to measure the things.

The chosen scale is φ for the great edge of the main faces. A right angle line joins from Ω to Ω, of the opposite faces of the polyhedron. This line crosses the common center of the two spheres.
THE SHERES

The chosen scale is φ for the great edge of the main faces. Whereby the radius of the spheres are:

\[ R = \frac{D_{ext}}{2} = \frac{\sqrt{(2\phi + 3)}}{2} \approx \frac{5}{4} \text{ at } 1.1 \% \]
\[ r = \frac{D_{in}}{2} = \frac{\sqrt{(2\phi - 1)}}{2} \approx \frac{3}{4} \text{ at } 3.1 \% \]

A simple equation expresses the relationship between \( r \) and \( R \):

\[ r^2 + 1 = R^2 = \frac{(2\phi + 3)}{4} \approx 1.559017 \]

This exact result is similar to the equation, equally true:

\[ \left(\frac{3}{4}\right)^2 + \left(\frac{4}{4}\right)^2 = \left(\frac{5}{4}\right)^2 \approx \frac{(2\phi + 3)}{4} \text{ at } 2.2 \% \]
THE WHOLE SOLID

De = \sqrt{(2\phi +3)} \approx 2,497 212 \approx \frac{5}{2} \text{ à } 1,1\% \text{°}

Di = \sqrt{(2\phi -1)} \approx 1,495 349 \approx \frac{3}{2} \text{ à } 3,1\% \text{°}

\frac{Di}{De} = \sqrt{[(2\phi -1)(2\phi +3)]} \approx 0,598 807 \approx \frac{3}{5} \text{ à } 2 \% \text{°}

The other informations are one the plan.
**HEXAGON AND HEIGHT**

- **Circle of Hexagon**
  \[ Dh = 2\sqrt{\frac{(2+\phi)}{3}} \approx 2,196.371 \]
  Minimal length (parallel to hexagon): \[ \sqrt{2+\phi} \approx 1,902.113 \]

- **Inclination of the panels/vertical** (yellow triangle):
  \[ \sin(\text{Alpha}) = \sqrt{\frac{(2+\phi)}{3}} \div (\phi+1) \approx 0.419469 \]
  => ‘Alpha’ \(\approx 24.801^\circ\)

- **Height of the polyhedron** (green triangle):
  \[ A = 2\sqrt{\frac{(2\phi+1)}{3}} \]
  then \(H = \left[\frac{L}{(\phi+1)}\right]A\) avec \(L = (1+\phi/2) + 1/2 = (\phi+3)/2\)
  
  • \[ ((\phi+3)-(\phi+1))^2 = (7\phi +10)-(3\phi +2) \]
  and \[ (2\phi +1)/3\cdot[(7\phi +10)-(3\phi +2)] = [(41\phi +24)-(3(3\phi +2))] \]
  So \(H = \sqrt{[(41\phi +24)-(3(3\phi +2))]} \approx 2,096.055\)

N.B.: The total height of the non-truncated rhombohedron is \(3\sqrt{\frac{(2+\phi)}{3}} \approx 3,564.875\)

*Comparative Geometry © Yvo Jacquier - The Polyhedron of « MELENCOLIA § I » - Dürer 7 on 10*
THE OBLIQUITY OF THE EARTH

Calculation of the angle

\[ \tan \beta = \frac{\cos \alpha}{R/r + \sin \alpha} \]

\[ \beta \approx 0,434 \]

\[ \beta \approx 23,482^{\circ} \]

\[ \beta \approx 23^{\circ} 28' 57'' \]

Comparison to the obliquity of the Earth

\[ \beta' \approx 23,438^{\circ} \]

\[ \Delta' \approx 0,045^{\circ} \]

\[ \Delta' \approx 2,7' \]

Tycho Brahé

\[ \beta'' \approx 23,514^{\circ} \]

\[ \Delta'' \approx 0,032^{\circ} \]

\[ \Delta'' \approx 1,9' \]
The sculpture is intended to express the lightness of the light on the fundamental structure of the polyhedron of Dürer. The designated material is the plexiglass, supported by some metals (stainless steel?). They support the development of lighting - key point.

A hexagonal basin filled with a few inches of water will act as a mirror. A dark color (dark blue?) will make this effect stronger.

- The first highlight of the sculpture is to make explicit the obliquity of the Earth in the structure of the polyhedron. The Ω points, contact of the inscribed sphere with the large faces, are from the pole of the large sphere at an angle of 23.5 ° with the vertical axis.

- The second strong point is to present all the lines of the polyhedron in materializing only two large faces. Just a few cables are needed to complete the design of the others.

Are materialized: the outer sphere, the inner sphere, the Ω point, the central axis of the spheres, a crescent moon - hung on the axis of the Earth relative to the ecliptic. Some secondary lines, engraved, may represent the pentagrams, circles etc.
THE ELEMENTS

LEXIQUE

1  Large circular plate, representing the circumscribed sphere
   This inclined plate is between the two points of the opposite faces.

2  Small vertical plate, representing the inscribed sphere. It passes through the first (slot)
   It is extended by two triangles that secure the large faces
   Top and bottom, two vertical tubes come to grip the plate into their slots (B2 et B3)

3  Large face of the polyhedron. Screwed on the small vertical plate.

3bis Opposite side, same fixation with the vertical plate.
   It is also fixed to the metal portion at the triangular base (Tb).

4  The crescent moon is a cross between two great circles
   It has several potential attachment points on its inner edge

B1  The main reinforcement bar adopts the perspective of the Earth: obliquity.
     It is based in the water on a solid plot
     It crosses the face 3bis, just beside the point Ω
     It is pasted to the vertical plate 2
     She joins the B2 bar across. Many mounting options

B2, 3 Bars supporting the plate 2

Ta  The equilateral triangle of the top points the left direction on the profile view
     It is integral with B1 and B2, and it hooks the side 3, on the side with the same width
     It is the anchor of two cable types C, which restore the empty faces

Tb  Equilateral triangle in water. Support of the sculpture.
     Its thickness is calculated for the large sphere touches the bottom of the water, vertically